SOFT MASSIVE SPRING

Objectives:

1) To determine the spring constant and the mass correction factor for the given soft massive spring by the static (equilibrium extension) method.
2) To determine the spring constant and the mass correction factor for the given soft massive spring by the dynamic (spring mass oscillations) method.
3) To determine the frequency of oscillations of the spring with one end fixed and the other end free, i.e. zero mass attached.
4) To study the longitudinal stationary waves and to determine the fundamental frequency of oscillations of the spring with both the ends fixed.

Apparatus:

1) A soft massive spring
2) A long and heavy retort stand with a clamp at the top end
3) A set of calibrated masses with hooks (including fractional masses)
4) A function generator with its connecting cord
5) A mechanical vibrator mounted on the retort stand
6) A digital stopwatch
7) A measuring tape
8) A tissue paper

Introduction:

A spring is a flexible elastic device, which stores potential energy on account of straining of the bonds between the atoms of the elastic material of the spring. A variety of springs are available which are designed and fabricated to suit the various mechanical systems. The most common types of springs are compression springs, extension springs and torsion spring. There are some special types of springs like leaf spring, V-spring, spiral spring etc. The coil or helical type of springs can have cylindrical or conical shape.

Robert Hooke, a 17th century physicist, studied the behavior of springs under different loads. He established an equation, which is now known as Hooke's law of elasticity. This law states that the amount by which a material body is deformed (the strain) is linearly proportional to the force causing the deformation (the stress). Thus, when applied to a spring, Hooke's law implies that the restoring force is linearly proportional to the equilibrium extension. $F = -K x$, where F is the restoring force exerted by the spring, x is the equilibrium extension and K is called the spring constant. (The negative sign indicates that the force F is opposite in direction to the extension x. Hence also the term 'restoring force'.) For this equation to be valid, x needs to be below the elastic limit of the spring. If x is more than the elastic limit, the spring will exhibit 'plastic behavior', wherein the atomic bonds in the material of the spring get broken or rearranged and the spring does not return to its original state. It may be noted that the potential energy U stored in a spring is given by $U = ½ K x^2$.

Depending on the value of the spring constant, a spring can be called as a soft or hard spring. A spring can be called massless or massive, depending on the mass, which needs to be attached to get a considerable extension in the spring. Sometimes springs are also categorized by the ratio of spring constant to the mass of the spring (K/m_s). A soft massive spring has a low spring constant and its mass can not be neglected.

In the determination of the spring constant of a spring, we generally neglect the effect of the mass of the spring on the equilibrium extension or the time period of oscillation of the spring for a given mass attached. In the case of soft massive springs, the mass of the spring cannot be neglected. These types of springs have extension under their own weight and therefore need a correction for the extension. Similarly, they oscillate without any attached mass, which implies that the standard formula for the time period of oscillations of a spring needs modification. People have theoretically worked out the modification and corrected the formula for the equilibrium extension and also the time period of oscillations. Interestingly, one finds that the mass correction factors in these two cases are not the same. In this problem, we will experimentally study and verify the modified formulae.

An extended soft massive spring clamped at both ends can be assumed to be a uniformly distributed mass system. It has its own natural frequencies of oscillation (corresponding to different normal modes) like a hollow pipe closed at both ends. Using the method of resonance, we will excite and study different normal modes of vibration of the spring. Here longitudinal stationary waves will be set up on the extended soft massive spring.

Description:

In Part A, we will use the static method, where the equilibrium extension of a given spring will be measured for different attached masses and the spring constant and the mass correction factor will be determined. In Part B, we will use the dynamic method, where different masses will be attached to the lower end of the spring with its upper end fixed and the corresponding time period of oscillations for such a spring-mass system will be measured. Also the frequency of oscillations of the spring with the upper end fixed and the lower end free, i.e. the zero attached mass, will be determined graphically. In Part C, we will use a mechanical vibrator to force oscillations on the spring and excite different normal modes of vibration of the spring. Thus the longitudinal stationary waves will be set up on the spring. We will measure the frequencies of excitation corresponding to different normal modes. From these, the fundamental frequency of oscillation with both the ends fixed will be determined. We will compare this frequency with the frequency of oscillations with one end fixed and the other end free as determined earlier in Part B.

Theory:

Part A:

Let L_o is the length of the spring when the spring is kept horizontal under no tension, m the mass attached to the free end of the spring, L_m the length of the spring when the mass is attached at its lower end, S_m the equilibrium extension of the spring for mass m, m_s the mass of the spring, K the spring constant and g the acceleration due to gravity.

Thus,

$$S_m = L_m - L_o \tag{1}$$

Note that the tension in the spring varies along the spring from $(m + m_s)g$ at the top to mg at the bottom.

We can write,

$$T(x) = (m + m_s)\, g - C\, g\, x$$

where, C is a constant of proportionality. $C = m_s/L_o$ and x is the distance from the top of the given point; x varies from 0 to L_o.

We can determine the expression for S_m, by taking extension of a small element of length Δx and integrating over the total length of the spring.
The final expression which we get is

$$S_m = \left(m + \frac{m_s}{2}\right)\left(\frac{g}{K}\right) \tag{2}$$

where, $(m_s/2)$ is called the mass correction factor (static case) m_{cs}.

Part B:

The expression for the time period of oscillations T for an ideal (massless) spring-mass system is given by,

$$T = 2\pi\sqrt{\frac{m}{K}}$$

In case of the soft massive springs, we cannot neglect the mass of the spring since these springs can oscillate without any attached mass. We thus need to modify the above expression for T. This can be done using the principle of conservation of energy, i.e. Potential Energy + Kinetic Energy = constant.

The modified expression, which we get is,

$$T = 2\pi\sqrt{\frac{m + \left(\frac{m_s}{3}\right)}{K}} \tag{3}$$

where $(m_s/3)$ is again the mass correction factor (dynamic case) m_{cd}. Note that the mass correction factors in Part A and Part B are different.

The corresponding frequency of oscillations f' is given by, $f' = \frac{1}{T}$

Part C:

The extended spring serves as a uniformly distributed mass system. It has its own natural frequencies like a hollow pipe closed at both the ends [Note that, both the ends of the spring may be taken to be fixed. The upper end is fixed in any case and the amplitude of the lower end is so small, as compared to the extended length of the spring that it can be taken to be zero].

The natural frequencies correspond to stationary waves; their wavelengths are given by

$$\lambda_n = \left(\frac{2}{n}\right)L, \quad n = 1, 2, 3, \ldots$$

Now, $\lambda_n f_n$ = velocity V_o of the waves on the spring, where f_n is the frequency of the longitudinal stationary waves set up on the spring; n = 1 is the fundamental, n = 2 the second harmonic and so on:

$$f_n = \frac{V_o}{\lambda_n} = \frac{V_o\, n}{2L} \qquad (4)$$

$$f_1 = \frac{V_o}{2L}, \qquad f_2 = \frac{V_o}{L},. \qquad \ldots\ldots\ldots, \qquad f_n = n\, f_1$$

This fundamental frequency f_1 in this case should be twice that of the fundamental frequency f_o' of the spring with zero mass attached to the spring.

Experimental Setup:

For Part A and B, you will need a soft massive spring, a retort stand with a clamp, a set of masses, a measuring tape / scales and a digital stopwatch.

For Part C, you will need a soft massive spring, a long and heavy retort stand with a clamp at the top end and a mechanical vibrator clamped near the base of the stand. We will also need a function generator. In this case, the soft massive spring should be clamped at the upper end on the long retort stand. The lower end of the spring should be clamped to the crocodile clip fixed at the centre of the mechanical vibrator. The lower end of the spring will be subjected to an up and down harmonic motion using the mechanical vibrator. It must be ensured that the amplitude of this motion is small enough so that the ends could be considered to be fixed.

Warning:

1) Do not extend the spring beyond the elastic limit. Choose 'thoughtfully' the value of the maximum mass that may be attached to the lower end of the given spring.
2) Keep the amplitude of oscillations of the 'spring-mass system' just sufficient to get the required number of oscillations.
3) Remember always to switch 'ON' the power supply to any instrument before applying the input to it.
4) Use the mechanical vibrator very carefully. You should not get hurt with the sharp edges/corners. Be extremely careful while clamping the lower end of the spring to the vibrator using the crocodile clip.
5) The amplitude of vibrations should be carefully adjusted to the required level using the voltage selection and amplitude knob of the function generator.

6) Use the measuring tape carefully to avoid any injury. The tape is metallic, and the edges are very sharp.

Procedural Instructions:

Part A:

(i) Measure the length L_o of the spring keeping it horizontal on a table in an un-stretched (all the coil windings of the spring touching each other) position.

(ii) Hang the spring to the 'clamp' fixed to the top end of the retort stand. The spring gets extended under its own weight.

(iii) Take appropriate masses and attach them to the lower end of the spring. (Choose the range of the mass carefully, keeping in mind the elastic limit.)

(iv) Measure the length L_m of the spring in each case. (If required, you may repeat each measurement two or three times and take the average.) Thus determine the equilibrium extension S_m for each value of mass attached.

(v) Plot an appropriate graph and determine the spring constant K of the spring and also the mass correction factor m_{cs}. (Take g = 980 cm/s^2 = 9.80 m/s^2). Use straight line fit to determine slope and intercept and error in the slope and intercept.

Question 1: State and justify the selection of variables plotted on X and Y axes. Explain the observed behavior and interpret the X and Y intercepts.

Table1: Measurements for static case

L_0 = …..(cm)		
Mass (gm)	L_m (cm)	$S_m = L_m - L_0$ (cm)

Report K and m_{cs} with appropriate error.

Part B:

(i) Keep the spring clamped to the retort stand.

(ii) Try to set the spring into oscillations without any mass attached. You will observe that the spring oscillates under the influence of its own weight.

(iii) Attach different masses to the lower end of the spring and measure the time period of oscillations of the spring mass system for each value of the mass attached. (Choose the masses carefully, keeping in mind the elastic limit). Measure time for 10 oscillations and get the time period by deviding with 10, totally measure it for three times for each mass and determine the average time period. Pull the mass a little to make the spring oscillate with each mass to record the time period.

(iv) Perform the necessary data analysis by plotting the graph and determine the spring constant K and the mass correction factor m_{cd} using the above data. Use straight line linear fit.

(v) Determine the time period (T_0) and calculate the frequency f_o' corresponding to zero mass attached to the spring from the graph. Using the Eq.

$$f_o' = \frac{1}{2\pi}\sqrt{\frac{K}{m_c}} \qquad (5)$$

and the value of K, determine m_c (the mass correction factor). Check whether this is equal to mass correction factor m_{cd} obtained using the graph in step (iv).

Question 2: Does the above method of measuring the total time for a number of oscillations help us to increase the reliability of time period measurement?

Table2: Measurements for dynamic case

Mass (gm)	Time period of chosen number of oscillations, T (sec)			Average T (sec)
	1st measurement	2nd measurement	3rd measurement	

Report K and m_{cd} with appropriate error.

Part C:

(i) Keep the spring clamped to the long retort stand.

(ii) Clamp the lower end of the spring to the crocodile clip attached to the vibrator.

(iii) Connect the output of the function generator to the input of the mechanical vibrator using BNC cable.

(iv) Starting from zero, slowly go on increasing the frequency of vibrations produced by the vibrator by increasing the frequency of the sinusoidal signal/wave generated by the function generator. At a particular frequency you will observe that the midpoint of the spring will oscillate with large amplitude indicating an antinode there. (You may use a small piece of tissue paper to observe the amplitude at the antinode.) This is the fundamental mode (first harmonic) of oscillation of the spring. Adjust the frequency to get the maximum possible amplitude at the antinode. Measure and record this frequency using the display on the function generator.

(v) Increase the frequency further and observe higher harmonics, identifying them on the basis of the number of loops you can see between the fixed ends.

(vi) Plot a graph of frequency versus the number of loops (harmonics). Determine the fundamental frequency f_o from the slope of this graph. Use straight line fit to determine the slope and intercept.

(vii) Compare this fundamental frequency f_o with the frequency f_o' of the spring-mass system with one end fixed and the zero mass attached (as determined in Part B) and show that $f_o' = (f_o/2)$.

Question 3: Explain why the two frequencies should be related by a factor of two? (Take the analogy between the spring and an air column.)

Table 3: Frequency and number of nodes

Frequency (Hz)	Number of loops (n)

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